RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College under University of Calcutta)

B.A./B.SC. FIFTH SEMESTER EXAMINATION, DECEMBER 2013

THIRD YEAR

PHYSICS (Honours)

Date : 16/12/2013

Time : 11 am – 1 pm

Paper : V

Full Marks : 50

Group – A

Sec – I

(*Answer* ***any three*** *of the following*)

1. a) For an N-particle constrained dynamical system, deduce d´ Alembert's principle, explaining clearly the assumptions made, if any. Use this principle to obtain the equation of motion of a simple pendulum. [4+1]

   b) A system has single generalised coordinate q with Lagrangian $L(q,\dot{q}) = \frac{\dot{q}^2}{4} - \frac{q^2}{9}$

   Consider the modified Lagrangian $L'(q,\dot{q}) = \frac{\dot{q}^2}{4} - \frac{q^2}{9} + 3q^2\dot{q}$ $\left(\dot{q} = \frac{dq}{dt}\right)$

   Show that L and L´ give the same equation of motion. Do they give the same Hamiltonian? Explain. [5]

2. a) What are the constants of motion of a dynamical system if the Lagrangian of the system has—
   i) translational symmetry,
   ii) rotational symmetry and
   iii) time-translational symmetry?

   Explain with examples in the respective cases. [6]

   b) If the kinetic energy T of holonomic system is a homogeneous quadratic function of the generalised velocities $\{\dot{q}_\alpha\}$, show that, $\frac{\partial T}{\partial p_\alpha} = \dot{q}_\alpha$, where p_α is the generalised momentum conjugate to q_α. [4]

3. a) The point of support of a simple plane pendulum moves vertically according to $y = \frac{1}{2}ft^2$, where f is constant.
   i) Write down the Lagrangian, taking as generalised coordinate the angle θ the pendulum makes with the vertical.
   ii) Obtain Lagrange's equation of motion and show that it is the same as that of a simple pendulum in a gravitational field (g+f). [5]

   b) A particle moves in the vertical plane (xz plane, say) with the z-axis along the direction of the vertical. The only force acting on the particle is the force of gravity. Write the Lagrange's equation of motion pointing out the existence of cyclic coordinates, if any.
   i) Find out the momenta (p_x, p_z) conjugate to (x,z).
   ii) Find out the Hamiltonian of the system. [5]

4. a) Define the Poisson Bracket of two dynamical variables u = u(q, p, t), v = v(q, p, t) Prove that—

   $\frac{du}{dt} = \frac{\partial u}{\partial t} + [u,H]_{P.B}$. for the variable u = u(q,p,t). If u and v are constants of motion show that $[u,v]_{P.B}$ is also constant. [5]

   b) Write down Euler's equation of motion of a symmetrical top is a torque free space. Use it to show that :
   i) the angular momentum vector has a constant magnitude
   ii) the angular velocity vector $\vec{\omega}$ precesses uniformly around the symmetry axis of the body. [5]

5. a) Two identical simple harmonic oscillates are coupled by a massless spring of spring constant K. The system lies along the x-axis. If the system is displaced slightly from equilibrium, obtain the normal frequencies for longitudinal oscillations, and the corresponding normal modes.

   Choose suitable normal coordinates Q_1,Q_2 (by inspection) and show that the Lagrangian in terms of these is the sum of two independent Lagrangian. [5+2]

   b) The Lagrange's equation of motion of a holoumic system can be obtained from the principle of least action. Using as the action, $S=\int L(q,\dot{q})dt$

   Show that if $S'=\int\left\{L(q,\dot{q})+\frac{df(q,t)}{dt}\right\}dt$ then $\delta S=0$ implies $\delta S'=0$, where the symbol δ has the usual meaning. What does this result signify in physical terms? [3]

Sec – II

(*Answer* ***any two*** *of the following*)

6. a) Write down the Lorentz transformation between two inertial frames with a relative velocity v along the common x axis. Show that the speed of light in free space remains invariant from these two frames. [4]

   b) What is the proper time interval? Find what this time interval will be from an inertial frame moving with a velocity v relative to the first one. Pions can be produced in an accelerator and are found to have a half life of $1{\cdot}77\times10^{-8}$ sec. A collimated beam of pions leaves an accelerator at a speed of 0·99c where c is the speed of light. Find the distance travelled over which half the pions in the beam decay on the average. [6]

7. a) Deduce the expression for the transformation of velocities between two inertial frames. Hence show that the speed of light in free space remains the same in all inertial frames. Explain how do you specify the mass of a photon if no rest frame exists for the photon? [4+2+1]

   b) A liquid of refractive index n is flowing with a uniform velocity v in a transparent cylindrical chamber. A beam of light is passing through the liquid in the same direction as the liquid flow. What will be the speed of light in the liquid as seen from the laboratory? Keep terms upto order $\left(\frac{v}{c}\right)^2$ in any approximation that you may have to do. [3]

8. a) i) Define time-like and space-like intervals. Show that events related by time-like intervals are always causally related.

   ii) Use four vector formulation to find expressions for the 4-velocity u and the 4-force F. Show that their scalar product is always zero. [4+4]

   b) If T be the kinetic energy and p be the magnitude of the three momenta of a particle, find the mass of the particle in term of them in the relativistic calculation. [2]

9. a) A light source in relative motion with an observer, emits light whose wave front can be taken as plane. The direction of emission makes an angle θ with the direction of relative motion. Find what will be the Doppler shifted frequency of the light seen by the observer. Show that the spectrum of light from a distant galaxy which is receding from us with a velocity v gets redshifted. Find the amount of the red shift. [6]

   b) A particle of mass M and at rest decays into two bodies of which one is of mass m and the other is massless. Show that, in the relativistic calculation the velocity of the daughter particle of mass m is given by $\left(\frac{M^2-m^2}{M^2+m^2}\right)c$. [4]